Department of Physics, IIT-Kanpur

Time : 2 hrs. **PhD Admission Test May 2017** **Total Marks: 70**

Question 1

(A) Consider the harmonic oscillator given by the Hamiltonian $H = \frac{p^2}{2} + \frac{x^2}{2}$

Define the operators $a = \frac{1}{\sqrt{2\hbar}}(p - ix)$ and $a^\dagger = \frac{1}{\sqrt{2\hbar}}(p + ix)$

(i) The ground state $|0\rangle$ satisfies $a|0\rangle = 0$. Use this to find the ground state wave-function $\psi_0(x)$; you do not have to normalize it. Also find the corresponding energy? **[4+2 marks]**

(ii) A perturbation $V(x) = \epsilon x^4$ is added to the Hamiltonian. Calculate the first order correction to the ground state energy due to $V(x)$. **[4 marks]**

Useful formulae:

$$\int_{-\infty}^{+\infty} \exp(-\alpha x^2)\, dx = \sqrt{\frac{\pi}{\alpha}} \quad ; \quad \int_{-\infty}^{+\infty} x^{2n} \exp(-\alpha x^2)\, dx = (-1)^n \frac{d^n}{d\alpha^n} \int_{-\infty}^{\infty} \exp(-\alpha x^2)\, dx$$

Question 2

(A) A spherical shell of radius R carries a surface charge distribution $\sigma(\theta) = \sigma_0 \cos\theta$ (standard notation of spherical coordinates is used).

(i) Write the expansion of the electrostatic potential $V(r, \theta)$ inside and outside the shell using Legendre polynomials.

(ii) Write the appropriate boundary conditions satisfied by the potential.

(iii) Solve for the potential everywhere. **[6 marks]**

You may find the following information helpful:

$$P_0(X) = 1; \quad P_1(X) = X; \quad P_2(X) = \frac{1}{2}(3X^2 - 1)$$

(B) An infinitely long cylinder of radius R carries a magnetization $\vec{M} = ks^2\hat{\varphi}$ (standard notation of cylindrical coordinates is used).

(i) what is the divergence of the auxiliary field $\vec{H} = \frac{1}{\mu_0}(\vec{B} - \vec{M})$, where $\vec{B}$ is the magnetic field due to the given magnetization?

(ii) Find the value of $\vec{H}$ and $\vec{B}$ everywhere. **[4 marks]**

Question 3

(A) A spherical ball of mass m falls under gravity in a viscous fluid. Find the *position* $x(t)$ and *velocity* $v(t)$ of the ball as a function of time t. Assume that the mass starts from rest at a height h above the ground at $t = 0$. Solve for $x(t)$ and $v(t)$ assuming a turbulent drag of the form γv^2, where γ is a constant and v is the velocity.. **[6 marks]**

(B) For a complex function

$$f(z) = \frac{z^2 - 1}{z}$$

perform the contour integral $\oint f(z)dz$ over a circle of radius 2 with its center at the origin. **[4 marks]**

Question 4

Consider a system with the Hamiltonian

$$H = \frac{p^2}{2} + \frac{q^2}{2} - 2q \cos 2t \cos 3t$$

of a particle of unit mass, generalized coordinate q, and generalized momentum p. Write the Euler-Lagrange equation for the system in terms of q and solve it for the initial condition: $q(0) = a$ and $\dot{q}(0) = 0$. **[10 marks]**

Question 5

(A) How many significant figures are there in the product of 0.007 and 1.2345? Express the product to the correct number of significant figures. **[2 marks]**

(B) What is the maximum percentage uncertainty in x, if $x = (23.381 \pm 0.007) - (23.178 \pm 0.006)$? **[2 marks]**

(C) A student wants to determine the acceleration due to gravity (g) by measuring the time-period of a simple pendulum. Determine the mean value and the sample standard deviation of the g values from the data given below. Calculate the best value of the uncertainty in g (standard deviation of mean). Uncertainties in the measurement of length of string and period of oscillation are not known. **[6 marks]**

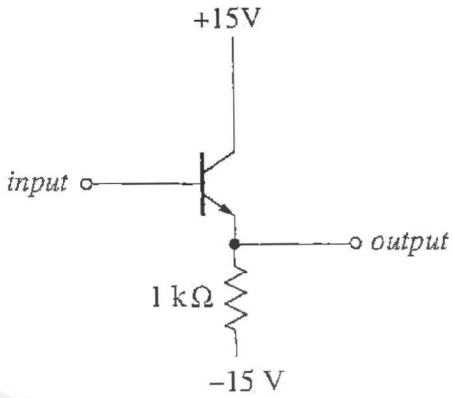
Length of string (in meter)	1.00	0.75	0.50
Period of oscillation (in seconds)	2.01	1.73	1.42

Question 6

(A) A transistor circuit is shown in the figure on the right.

(i) If the input voltage = 0.2 V what will be the output voltage? **[2 mark]**

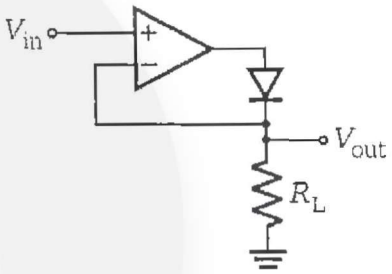
(ii) If the input voltage can be expressed as $V_{in} = 2sin(20t)$ (with t in seconds and V_{in} in volts) write an expression for the output voltage. **[3 marks]**



(B) In the opamp circuit shown, assume that the opamp is powered from ±15 V supplies. $R_L = 1.0$ kΩ. **[5 marks]**

(i) What is the output of the circuit if $V_{in} = +5$ V?

(ii) What is the output of the circuit if $V_{in} = -5$ V?



Question 7

For a system of non-interacting electrons at temperature T and chemical potential μ :

(A) Show that the probability of finding an electron in a state with energy δ above the chemical potential is the same as the probability of finding a hole at energy δ below the chemical potential. **[4 marks]**

(B) For the above system, suppose that the density of states $g(\varepsilon)$ is given by :

$$g(\varepsilon) = \begin{cases} \sqrt{(\varepsilon - \varepsilon_o)}\ , & \text{for } \varepsilon > \varepsilon_o \\ 0\ , & \text{for } \varepsilon_o > \varepsilon > 0 \\ \sqrt{(-\varepsilon)}\ , & \text{for } \varepsilon < 0 \end{cases}$$

where ε is the energy of the electron and ε_o is a constant. Find the value of the chemical potential writing all steps clearly. (At $T = 0$ electrons occupy states up to $\varepsilon = 0$. However, at finite T some electrons are excited to higher energies). **[6 marks]**