

Department of Physics, IIT-Kanpur

Time : 2 hrs.

PhD Admission Test May 2014

Total Marks: 100

Q1a) Consider an electron with charge $q = -e$ at rest in presence of a magnetic field $\mathbf{B} = B\hat{z}$.

i) Write down the Hamiltonian in matrix form (neglect the orbital motion). Obtain the energy eigenvalues and the corresponding spinors. [3]

ii) Write down the time-evolution operator for this system. Assuming that at $t = 0$ the spinor is given by

$$\xi(t=0) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Find the spinor at a later time t i.e. $\xi(t)$. [4]

iii) What is the probability of finding the electron with $\langle S_x \rangle = -\hbar/2$ at a later time t ? [3]

iv) Calculate the average value of S_z at a later time t . [2]

Q1b) Consider a simple harmonic oscillator whose quantum mechanical Hamiltonian is given by

$$H = \frac{\epsilon}{2} \left[-\frac{d^2}{d\eta^2} + \eta^2 \right],$$

where ϵ is the energy unit and η is the dimensionless position coordinate. Its orthonormal eigenket is represented by $|n\rangle$ and the raising and lowering operators are, respectively,

$$b^\dagger = \frac{1}{\sqrt{2}} \left[\eta - \frac{d}{d\eta} \right], \quad b = \frac{1}{\sqrt{2}} \left[\eta + \frac{d}{d\eta} \right].$$

i) Using $|n\rangle, b, b^\dagger$ and their various properties, calculate $\langle n' | \eta | n \rangle$. [2]

ii) Using the previous result, write down the matrix representation of the position operator η . [3]

iii) Calculate $\langle n | \frac{d^2}{d\eta^2} | n \rangle$. [3]

Q2a) Consider an invertible matrix A

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

Find the eigenvalues and the corresponding normalized eigenvectors.

Now consider another matrix B

$$B = ADA^{-1}$$

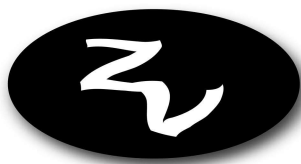
where D is a diagonal matrix

$$D = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

Find the eigenvalues and eigenvectors of B .

[8 + 2]





Q2b)

i) Show that in an arbitrary central force field the orbit of a particle will be in a two dimensional plane. [2]

ii) Consider a central force field where the potential is given as

$$V(r) = -\frac{A}{r^n}$$

where A is some arbitrary positive constant.

Write down the one-dimensional effective potential in radial direction.

Show that for ($n < 2$), the system admits a stable circular orbit.

Comment on the existence and the stability of circular orbits for ($n \geq 2$) [2 + 3 + 3]

Q3a) A particle of mass M and charge Q starts from the origin ($x=y=z=0$) with a speed V . The velocity has equal components in all the three directions. There is a uniform magnetic field $\vec{B} = B_0\hat{z}$, and an acceleration due to gravity along $-\hat{z}$ direction.

(i) Plot the displacement along z direction as a function of time.

(ii) Sketch the trajectory of the particle.

(iii) How many rotations the particle would make by the time its displacement along z axis becomes zero.

(iv) Estimate the trajectory length covered by the particle by that time. [3 + 3 + 3 + 3]

Q3b) Consider an electric dipole $\vec{p} = p_0\hat{z}$ located at $\vec{r}_0 \equiv (0, 0, 1)$, in an external potential $V_{ext}(\vec{r}) = a_0r^2$. Find the energy of the dipole, and the force on the dipole. [4 + 4]

Q4a) Consider N noninteracting particles moving in a large three-dimensional enclosure of volume V . The energy of a particle depends on the magnitude of its momentum, $\epsilon(\vec{p}) = v_0|\vec{p}|$. The enclosure is in thermal equilibrium through a thermal contact with a reservoir at temperature T . Find (i) the equation of the state (ii) the internal energy and (iii) the specific heat of the system. [5 + 4 + 1]

Q4b) Consider a plane monochromatic optical beam, linearly polarized, incident on a Young's double slit. The distance between the two slits is d . The width of the slit is negligible compared to d and the wavelength of the optical field, λ .

i) Choosing the middle of the slits as the origin of your co-ordinate system, estimate the location of the second principal maxima on a screen at a distance L from the slits. [2]

ii) If the incident intensity of the optical field is I_0 , what is the intensity at the first principal maxima? [2]

iii) Draw the resulting intensity pattern as a function of the position on the screen. [2]

iv) Now suppose one of the slits is blocked with a quarter wave plate, with the axis oriented along the direction of the incident polarization. This generates circular polarization. Draw the resulting intensity pattern, as observed on the screen. [4]

Q5a) A square pulse of height $5V$ and pulse width of 1ms is given as input to the following two circuits. For each case, give the circuit, write down the expression for the output voltage, and give a qualitative sketch of the output waveform as would be seen on an oscilloscope :

i) A 0.1H inductor connected in series to $10\ \Omega$ resistor which is grounded on the other side. The input voltage is given across ground and the inductor, and the output voltage is measured across the resistor.

ii) A differentiator using an ideal operational amplifier, with values of your choice for the components you use. Justify the values of components. [5 + 5]

